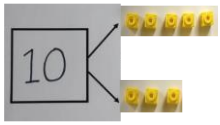
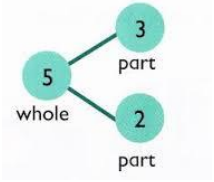
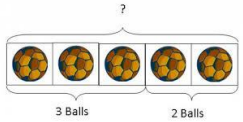
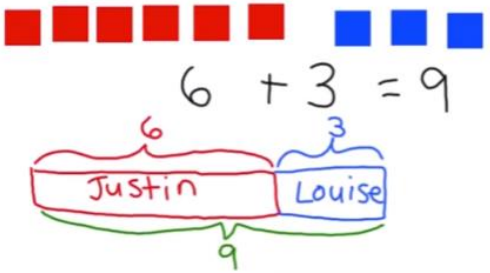
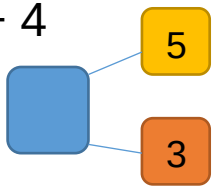
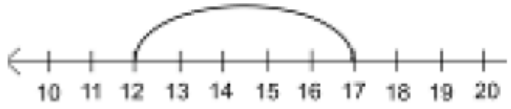
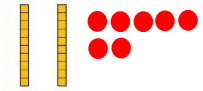
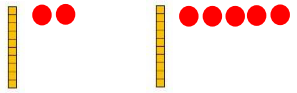
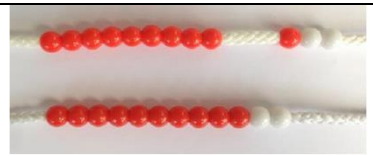
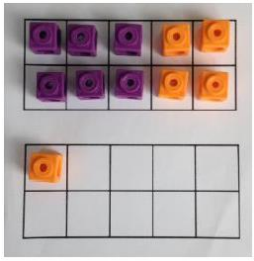
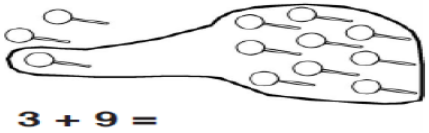
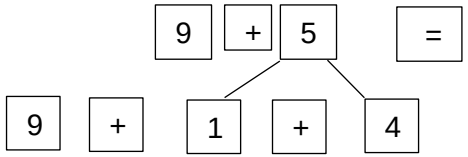
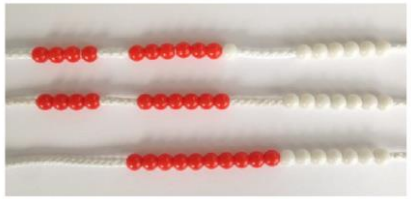
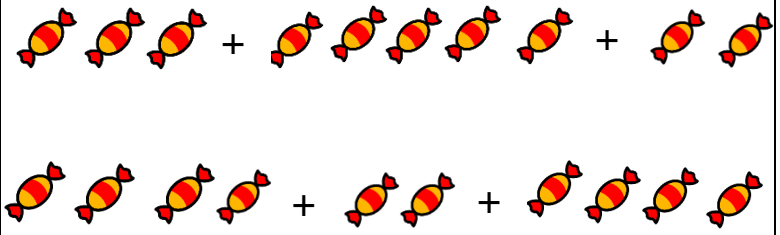
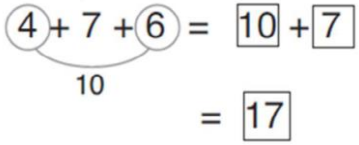
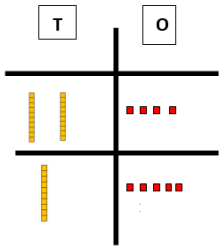
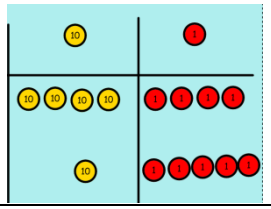
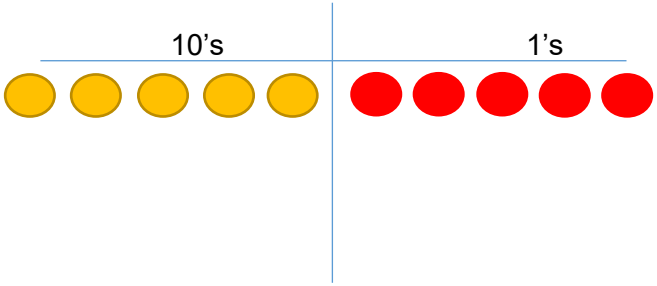
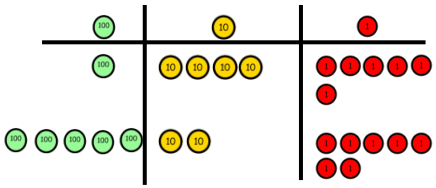
Progression in Calculations

Addition

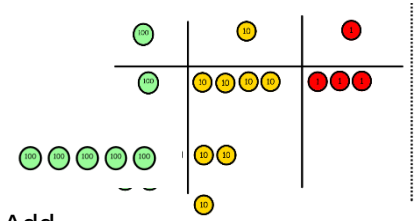
Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	   Use cubes to add two numbers together as a group or in a bar.   Use Numicon to represent the sum.	   Use pictures to add two numbers together as a group or in a bar.  Justin made 6 cookies. Louise made 3 cookies. How many cookies did they make in all? 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.

Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer. Also use blank number lines on whiteboards	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Partitioning to add		 = 27  12 + 15	10 2 + 10 5 20 + 7 12 15 10 2 10 5
Regrouping to make 10	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. **Complete activities reasoning with egg boxes cut into two rows of five**	Use pictures or a number line. Regroup or partition the smaller number to make 10. **Understanding built with frequent use of egg boxes and represent on a number line both numbered and blank**  3 + 9 = 10 + 2 = 12  	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now? **Use a range of representations of numbers using a range of equipment**

Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	 Combine the two numbers that make 10 and then add on the remainder.
Column method - no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  Represent base 10 counters like this: = 10 ◯ =1 	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 42 + 21 (40) + 20) = 60 (2 + 1) = 3 60 + 3 = 63 Or 42 + 20 = 62 + 1 = 63
Column method - regrouping	Make both numbers on a place value grid. 146 + 527	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition.



Add up the units and exchange 10 ones for one 10.



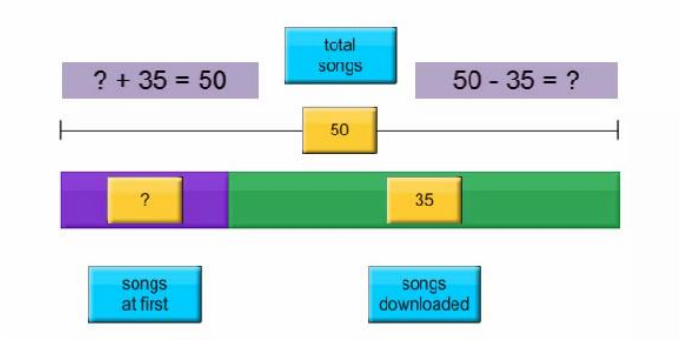
Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

1000's	100's	10's	1's
●● ●●●●	●●●●● ●●●●●	●●● ●	●●●● ●●●●● ●●
7	1	5	1
●		●	

Another visualisation technique is the part/part whole model – using blocks to visualise numbers and their relationship to each other.
Emma had some songs on her MP3 player, she downloaded 35 songs. Emma has 50 songs how many songs did she start off with?



$$\begin{array}{rcl} 20 & + & 5 \\ 40 & + & 8 \\ \hline 60 & + & 13 = 73 \end{array}$$

$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 1\ 1 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and

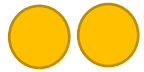
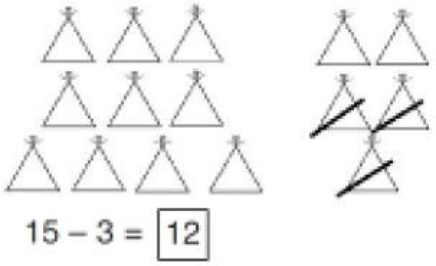
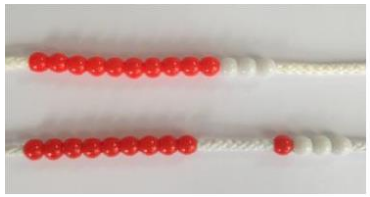
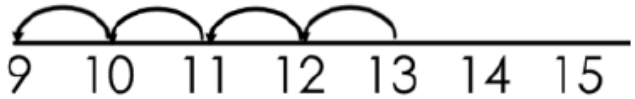
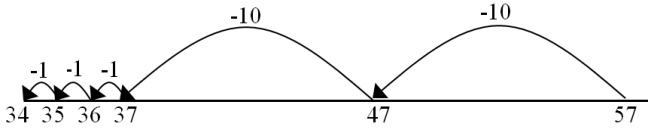
$$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ 1\ 1 \end{array}$$

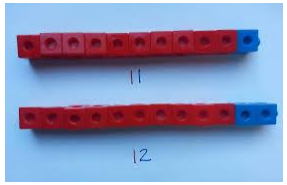
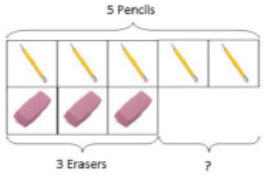
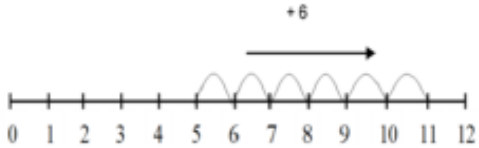
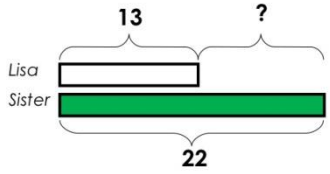
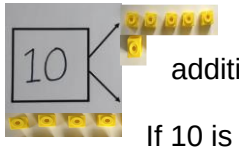
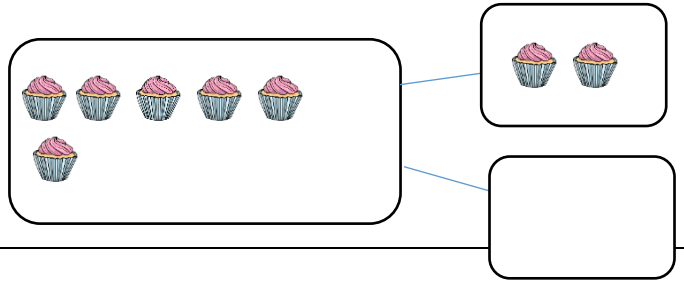
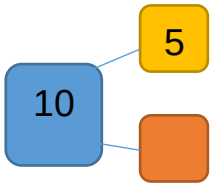
$$\begin{array}{rr} & £23.59 \\ + & £7.55 \\ \hline & £31.14 \\ & \ 1\ 1\ \ 1 \end{array}$$

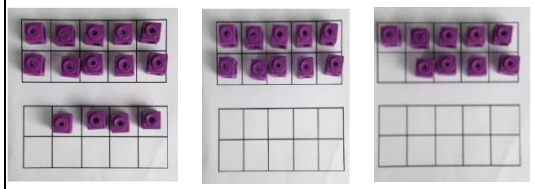
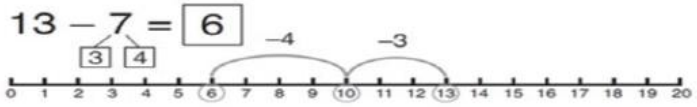
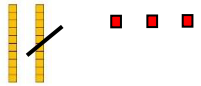
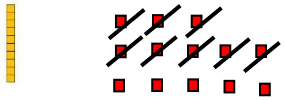
$$\begin{array}{rr} & 23.361 \\ & 9.080 \\ & 59.770 \\ + & 1.300 \\ \hline & 93.511 \\ & 2\ 1\ \ 2 \end{array}$$

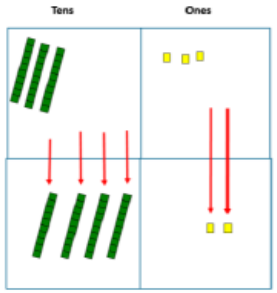
different. Money can be used here.

Subtraction

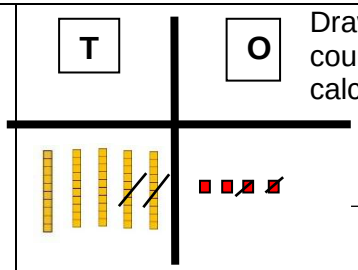
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  4 – 2 = 4  	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Partitioning		15 – 12  = 3	10 5 - 10 2 0 3 = 3
Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Part Part Whole Model	Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model. 	 Move to using numbers within the part whole model.

Make 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Developing mastery of partitioning numbers is very important to allow pupils to partition ‘creatively’ as the problem requires. 23-18 = 15  Exchange 10 into ones 	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

<table>
<tr>
<td>Column method without regrouping</td>
<td> 

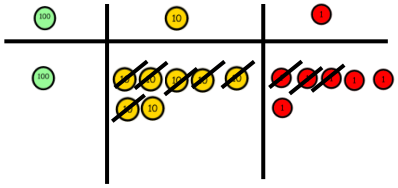
Use Base 10 to make the bigger number then take the smaller number away.

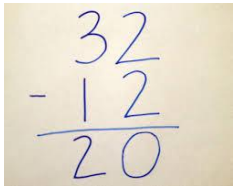
Show how you partition numbers to subtract. Again make the larger number first.</td>
<td> 

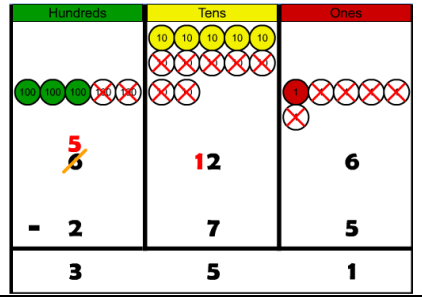
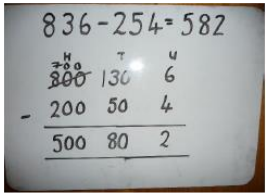
Draw the Base 10 or place value counters alongside the written calculation to help to show working.

54
-22

176
- 64
112

  </td>
<td>47 − 24 = 23
40 + 7
− 20 + 4
20 + 3

This will lead to a clear written column subtraction.
 
</td>
</tr>
<tr>
<td>Column method with regrouping</td>
<td>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters</td>
<td> 
</td>
<td> 
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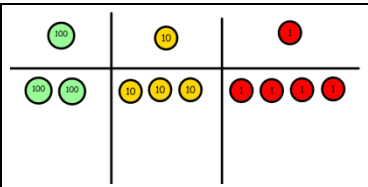
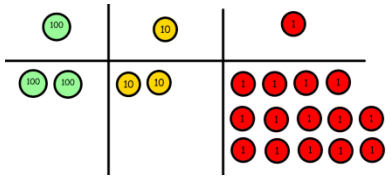


Calculations

234
- 88

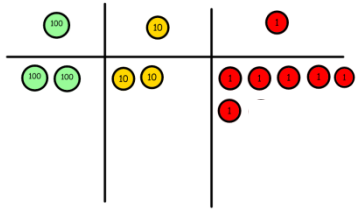
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

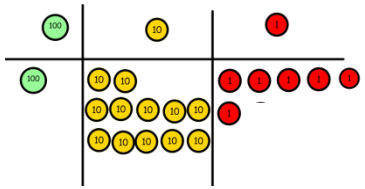
Now I can subtract my ones.



Calculations

234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



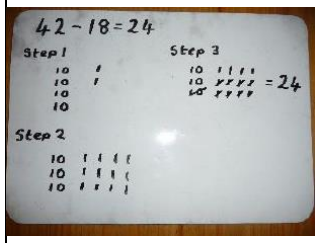
Calculations

234
- 88

Now I can take away eight tens and complete my subtraction

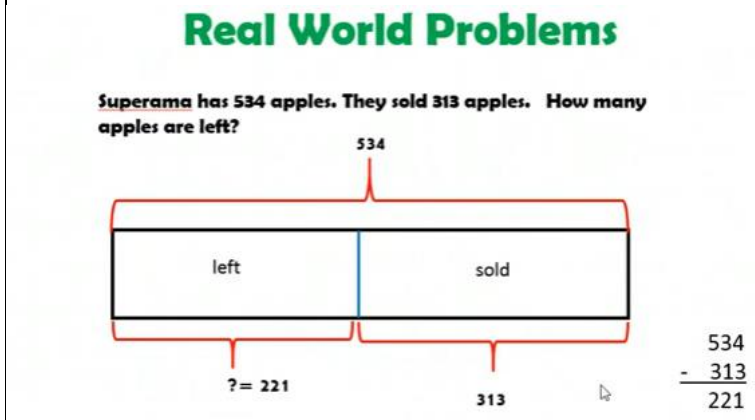
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

When confident, children can find their own way to record the exchange/regrouping.

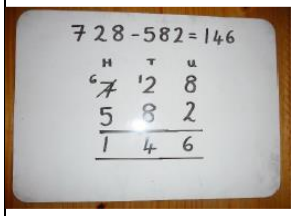


Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

Bar modelling subtraction



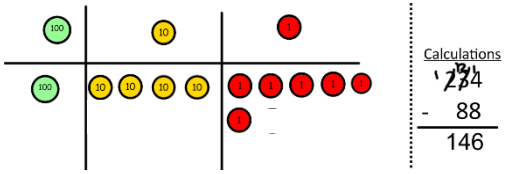
Children can start their formal written method by partitioning the number into clear place value columns.



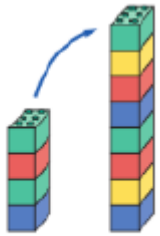
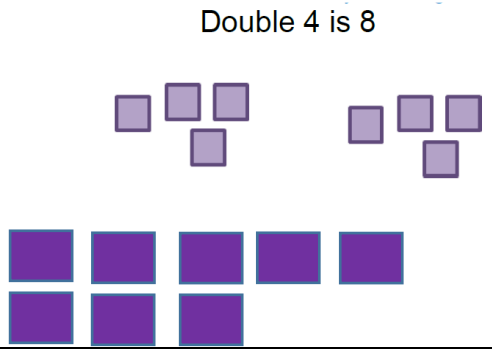
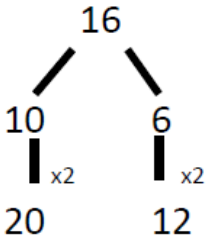
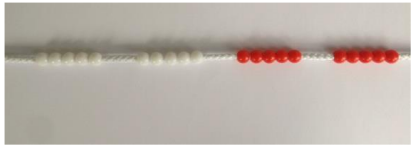
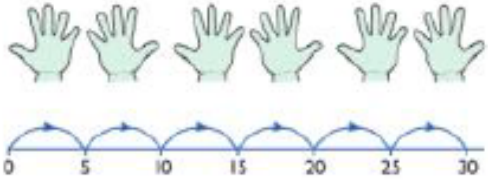
Moving forward the children use a more compact method.

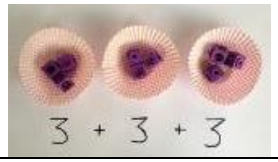
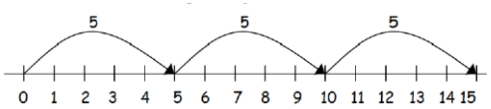
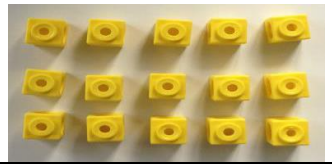
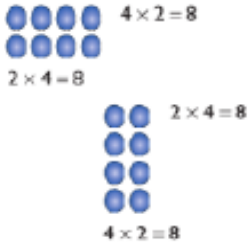
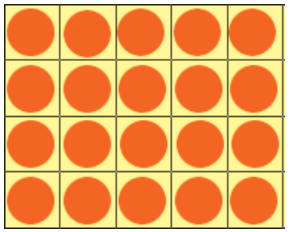
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} {\scriptstyle 5\ \ 12\ \ \ \ 1} \\ 2\ \not{6}\ \not{3}\ .\ \mathbf{0} \\ -\quad 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$

	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

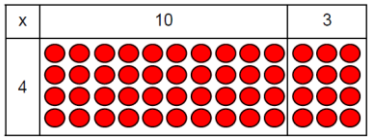
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition	  Use different objects to add equal groups. 	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures. 
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

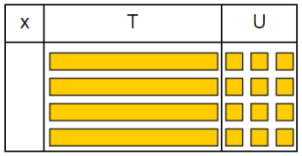
Grid Method

Show the link with arrays to first introduce the grid method.



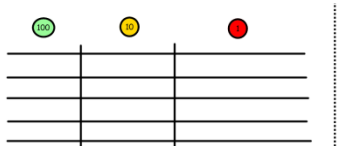
4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.



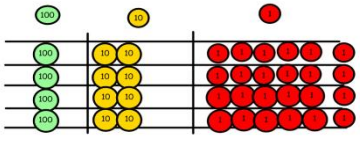
4 rows of 13

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Calculations
4 x 126

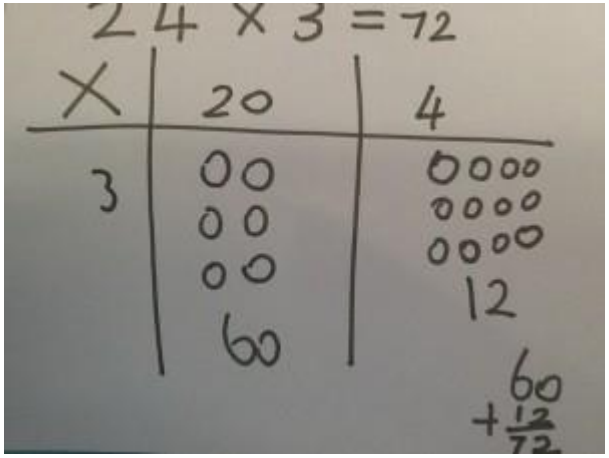
Fill each row with 126.

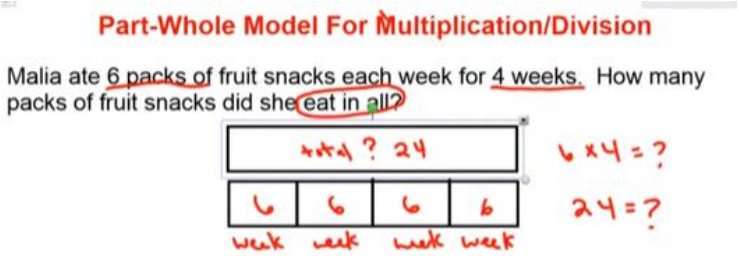


Calculations
4 x 126

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.





Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

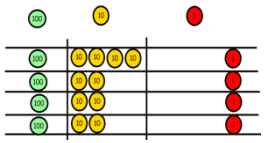
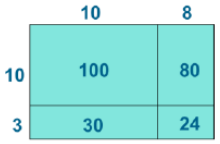
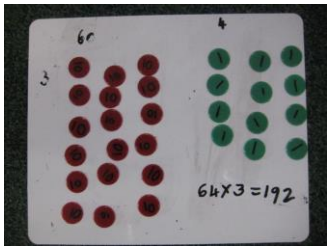
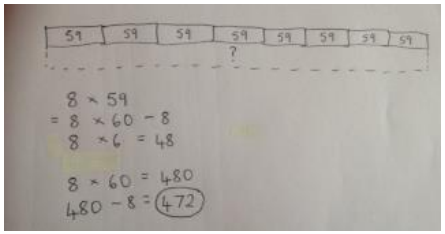
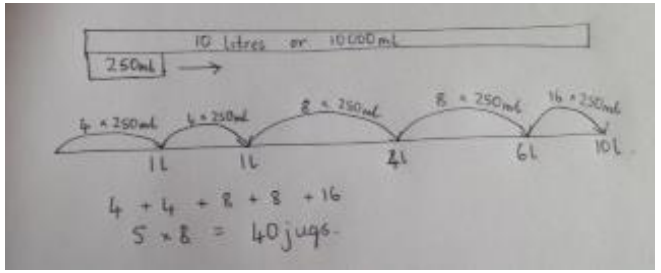
×	30	5
7	210	35

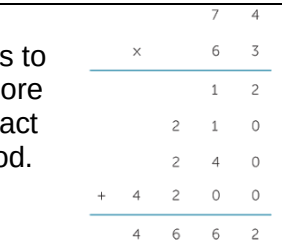
210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

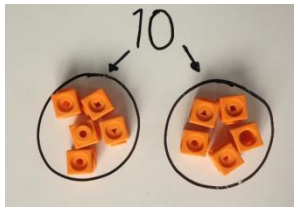
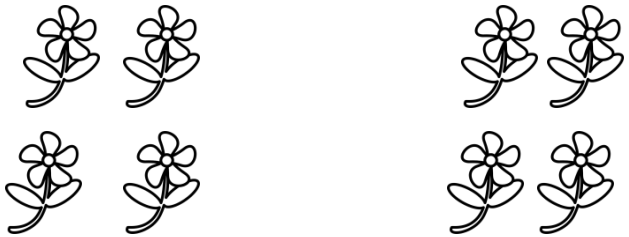
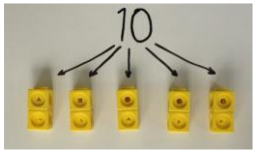
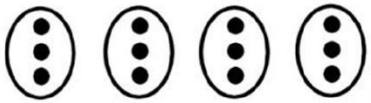
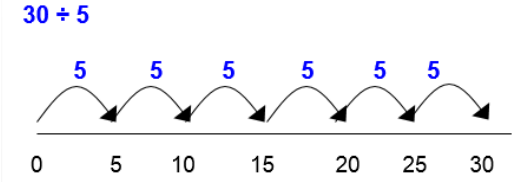
x	4
200	800
70	280
6	24
	1104

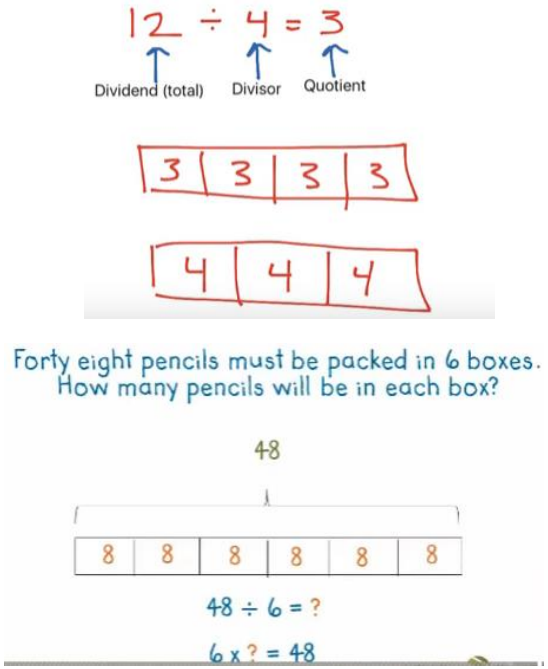
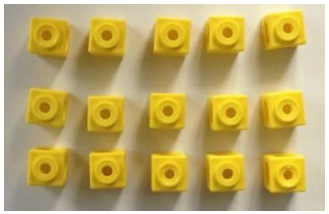
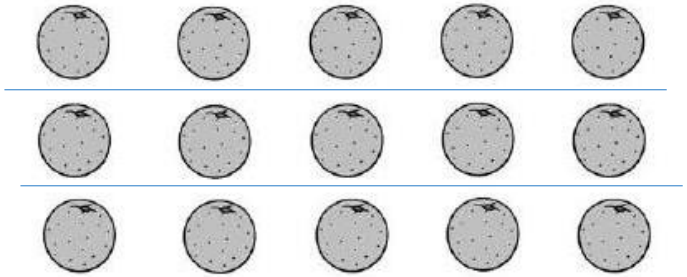
x	10	8
1000	1000	8000
300	3000	2400

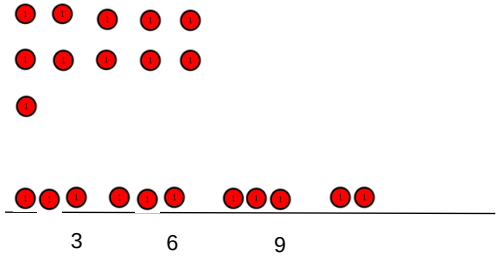
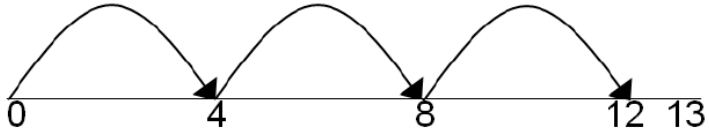
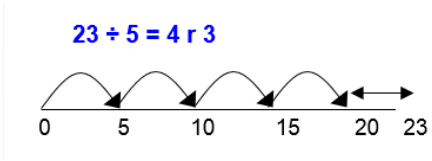
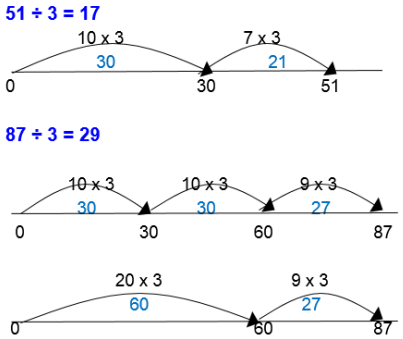
	Add up each column, starting with the ones making any exchanges needed.   Then you have your answer.		<table><tr><td>40</td><td>400</td><td>320</td></tr><tr><td>2</td><td>20</td><td>16</td></tr></table> 
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (2 x 4) 120 (30 x 4) 40 (2 x 20) 600 (30 x 20) 768

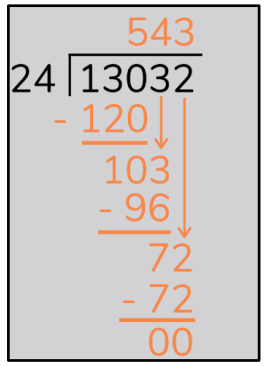
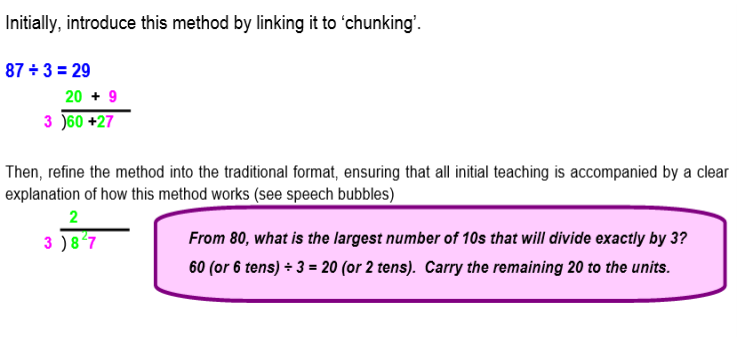
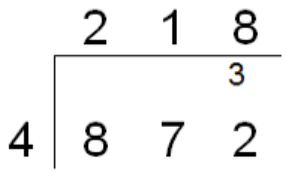
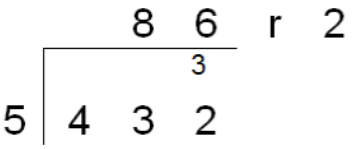
			This moves to the more compact method.  1342 x 18 10736 13420 24156

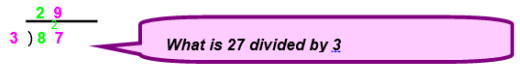
Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 10 ÷ 2 = 5 10 divided into groups of 2 equals 5 groups of 2  35 ÷ 5 = 7 Use a bead string and place pegs to show groups. Record the  repeated subtraction on a number line	*Draw dots to show an amount 12÷3 = 4*  *Use a number line to show jumps in groups. The number of jumps equals the number of groups.* 	28 ÷ 7 = 4 Divide 28 into groups of 7. How many groups of 7 can 28 be divided into? **Show the link between multiplication and division by exploring fact families from an early stage. Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.**

		Using the bar model to show division calculations 	
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

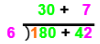
Division with a remainder	11 ÷ 3 = 3 r 2 Divide objects into groups of 3. How many complete groups and how many remain.  	Draw dots and group them to divide an amount and clearly show a remainder.  Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.   	Complete written divisions and show the remainder using r. 
Long division	51 ÷ 3 =  Bead strings 	Link long division first with a number line  Emphasise ‘efficient chunking’	Moving onto more formal methods of division. The number line method can be translated into the more formal long division method but pupils will need to be able to complete column subtraction confidently and know their times tables and related facts eg 3x4=12 30x4=120 etc. ‘Chunking’- division by subtracting ‘chunks’ of the divisor **132 ÷ 5 =** 26 r 2 5) 132 50 10 82 50 10 32 30 6 2 Then move onto a more refined and efficient ‘formal’ long division

		Before beginning a calculation pupils will be encouraged to create a list of facts using mental strategies. 1 x 24 = 24 2 x 24 = 48 3 x 24 = 72 4 x 24 = 96 5 x 24 = 120 10 x 24 = 240 This list will be written in a simplified to allow for speed of calculation e.g. 1 x 24 2 x 48 Pupils will be encouraged to use the most appropriate method for the division sum either long or short. Short division may be used for 2 digit numbers such as 11,12, 15 as well as single digits	
Short division		Short division can be shown as a method in itself as long as the pupils have a strong understanding of place value and have a strong understanding of 'fact families' where multiplication and division facts are linked. 	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder. Show the remainder as a fraction eg 86 2/5 

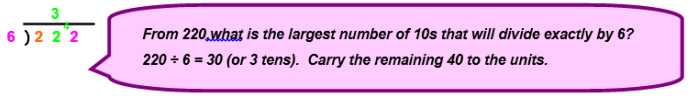


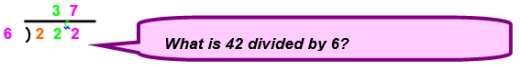
When this method is secure for TU ÷ U then quickly progress to HTU ÷ U
Again, begin by briefly linking the method to 'chunking', using numbers where there is no carrying in the hundreds.

222 ÷ 6 = 37



Refine the method, whilst clearly explaining the place value understanding.

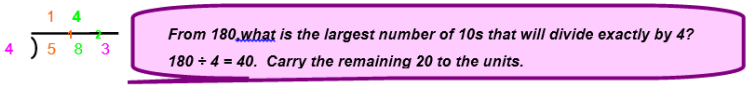




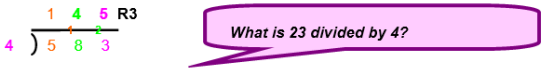
Finally, introduce examples of HTU ÷ U where there are also hundreds that need carrying, and where there are remainders.

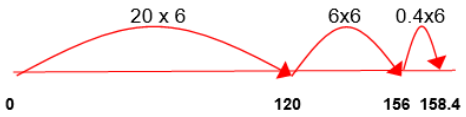
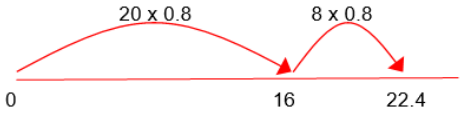
583 ÷ 4 = 145 r 3

Or, 'How many 4s in 500? The answer must be a multiple of 100.



Or, 'How many 4s in 180? The answer must be a multiple of 10.



Division of decimals		**158.4 ÷ 6 = 26.4**  **22.4 ÷ 0.8 = 28**  	Finally move into decimal places to divide the total accurately **188 ÷ 8 =** **2 3.5** **8) 1 8 8.4 0**